FINDING PLANS FOR REARRANGING ROBOTS IN $\theta$-LIKE ENVIRONMENTS

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Path Planning for Multiple Robots

- **Input:** Graph $G=(V,E)$ and a set of robots $R = \{r_1, r_2, ..., r_\mu\}$, where $\mu < |V|$
  - Each robot is placed in a vertex (at most one robot in a vertex)
  - A robot can move into an unoccupied vertex through an edge (no other robot is allowed to enter the vertex)
  - Initial positions of robots ... simple function $S_0: R \rightarrow V$
  - Goal positions of robots ... simple function $S^+: R \rightarrow V$

- **Task:** Find a sequence of allowed moves for robots such that all the robots reach their goal positions starting from the given initial positions
Motivation for the Problem

- Rearrangement of agents in tight space
- Automated control of heavy traffic
- Data transfer with limited size of the cache memory
A SPECIAL CASE WITH θ-LIKE ENVIRONMENT (θ-LIKE GRAPH)

- $G_\theta(a,b,c) = (V,E)$
- $\mu = |V| - 1$
- Relation to general graphs
  - Decomposition to bi-connected components
  - Cycle decomposition of a bi-connected component
  - Last cycle with a handle represents a θ-like graph

Example: Exchanging robots $r_1$ and $r_3$ (transposition of robots)

PlanSIG 2008

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THEORETICAL FOUNDATIONS FOR A SOLVING METHOD

- Interpret arrangement of robots as a permutation

- Proposition 1
  - Any permutation over $\mu$ elements can be obtained as a composition of at most $\mu-1$ transpositions.

- Proposition 2
  - Any even permutation over $\mu$ elements can be obtained as a composition of at most $\mu-1$ rotations along a triple.

- Proposition 3
  - Rotation along a triple is always solvable in a $\theta$-like graph; transposition is solvable, if the $\theta$-like graph contains an odd cycle.
A Solving Method – Theta-BOX

- **Pre-calculate** off-line optimal solutions for transpositions and rotations along triples
- **Compose a sub-optimal solution** of the optimal solutions of special cases
  - Use a fast alternative method if pre-calculated solutions are not available
  - Produces solutions of higher quality (**shorter**) than existing methods (Kornhauser et al., 1984 – MIT method)

[Graphs showing solving time and solution length comparisons between MIT and Theta-BOX]
CONCLUSIONS AND FUTURE WORK

- Special case of **path planning** for multiple robots in **θ-like** environments

- **Compose a sub-optimal solution** of the optimal solutions of sub-problems (Theta-BOX method)

- Produces **shorter solutions** than existing method

- **Future**: Improve a process of the search for optimal solutions of the special cases (transposition, triple rotation), currently – a variant of **IDA*** is used